# A Whimsical Principle and Physical Fundamentals

Jason Payne (潘傑森)

NCU Fundamental Theory Group Journal Club
October 11, 2017

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I: Relativity and Stability

II: The Necessity of Entanglement

## Overarching Themes

"We will trace two fairly disparate paths emanating from a lone goal: uncovering what physical theories can possibly lie beyond the current boundary of knowledge."

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"We will trace two fairly disparate paths emanating from a lone goal: uncovering what physical theories can possibly lie beyond the current boundary of knowledge."

"These days there seems to be nowhere left to explore, at least on the land area of the Earth. Victims of their very success, the explorers now pretty much stay home."

– Carl Sagan, *Pale Blue Dot: A Vision of the Human Future in Space*

- Re-evaluating even our most cherished physical notions, in particular the notion of "space" itself, as a means of clarifying what we know;
- "Classicalization," as opposed to quantization, as the appropriate perspective on the relation between the two.

# Part I: Relativity and Stability

## Insights Gleaned from our Quantum Relativity Project

⋆ **Naive Observation:** Physics is, fundamentally, a study of motion. Consequently, as we already saw in the infancy of our physics education, the notion of a “reference frame” is indispensable in describing motion. In this way, some manner of *Principle of Relativity* is already added to our physical lexicon at a decidedly elementary stage.

It therefore seems, at least at a very superficial level, that this principle might play a genuinely vital (perhaps even ubiquitous) role in determining breeds of physical theories exist, as well as how they behave.

**Question:** How seriously can one take this naive observation?

# The Unexpected Final Gift of *Relativity*

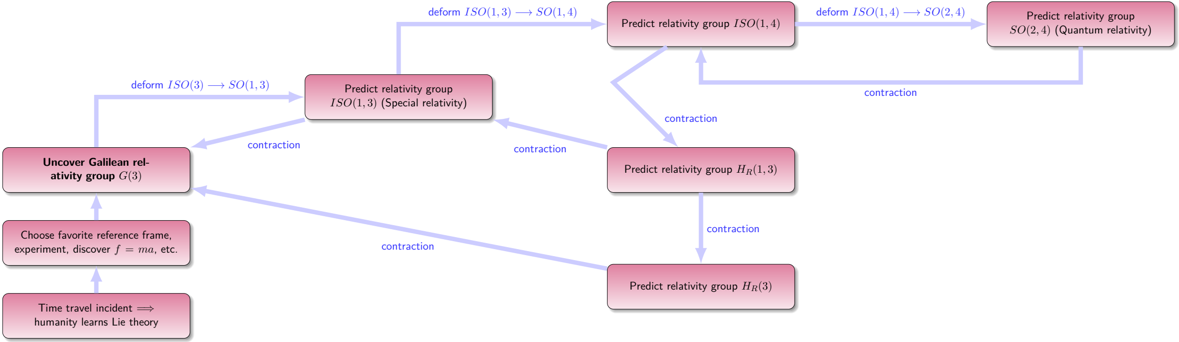
Fortunately for this endeavor, underneath the *Principle of Relativity* lies a rich mathematical structure.

- The operation of changing between admissible references frames clearly transforms the collection of such frames into a Lie group;
- There is a nice, familiar method of determining what other Lie groups are approximated by a given one! We can talk about deformations;

$$[X,Y] \longrightarrow [X,Y]_t = [X,Y] + td_1(X,Y) + t^2 d_2(X,Y) + \cdots$$

- Whether or not one should deform this group can be guided by the physically motivated notion of "stability" [5];
- We can also uncover further approximations by exploring the available contractions of a given Lie group (morally, just the reverse of deformations)!

# An Alternative History of Physics



# Who Needs a Panoply of Relativity Theories?

There is still a lesson to be learned from history:

> The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.
>
> – Hermann Minkowski, *Raum und Zeit*, Physikalische Zeitschrift, 10, 75-88, (1908)

$\star$ This statement reveals something of tremendous importance: our choice of relativity group directly alters the geometry of the physical space we populate! *How?*

# From Relativity to Models of Physical Space I

## The Relativity Group as a Manifold

The fact that our relativity group $G$ is a Lie group means that it already comes *equipped* with a geometry. So a first guess on how to obtain a model of physical space might be to use this directly (written at the Lie algebra level):

$$g'\cdot g = \exp(a'^j X_j)\exp(a^k X_k) = \exp\left(a'^j X_j + a^k X_k + \frac{1}{2}a'^j a^k [X_j, X_k] + \cdots\right) = \exp\left(\tilde{a}^l X_l\right)$$

In other words, we can imagine (at least in a neighborhood of the identity) the element $g'$ of our relativity group as taking the vector $\mathbf{a}$ to $\tilde{\mathbf{a}}$.

But... is this *really* what we are looking for?

# From Relativity to Models of Physical Space II

## The Coset Space Representation

Looking at any one of your favorite examples will immediately lead you to conclude the answer to the above question is "no."

Here, yet again, we can look at the structure of the relativity group to give us a hint: if the relativity group satisfies $G = G' \rtimes H$, then $H$ is *already* being thought of as acting on $G'$; hence $G'$ is really what we're after! We should quotient out this extra stuff!

In other words, take $X \in \mathfrak{g}$ ($=$ the Lie algebra of $G$) and $Y \in \mathfrak{G} := \mathfrak{g}/\mathfrak{h}$, then the analogue of the above equation

$$g' \cdot (gH) = (\exp(X)\exp(Y))H = \exp\left(X + Y + \frac{1}{2}[X,Y] + \cdots\right)H$$

can be interpreted $g'$ sending the vector $(\mathbf{a}, 1)^T \in \mathcal{G}$ to $(\tilde{\mathbf{a}}, 1)^T$. And this *works*!

# From Relativity to Models of Physical Space III

## A Lightning Fast Example: Special Relativity

Consider $G = ISO(1,3) = \mathbb{R}^{1,3} \rtimes SO(1,3)$. Taking

$$X = -\frac{i}{\hbar}(\omega^{\mu\nu} J_{\mu\nu} + a^\mu E_\mu) \in \mathfrak{iso}(1,3) \quad \text{and} \quad Y = -\frac{i}{\hbar} t^\rho E_\rho \in \mathfrak{iso}(1,3)/\mathfrak{so}(1,3),$$

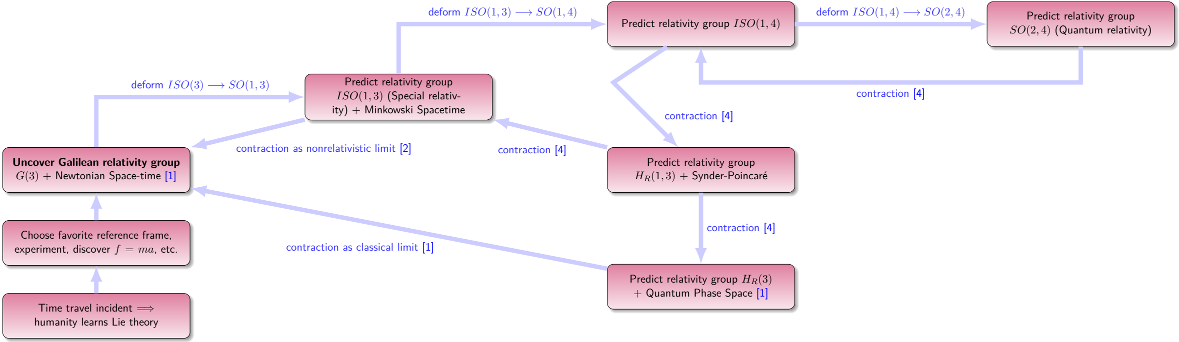
we obtain

$$\exp\left(-\frac{i}{\hbar}(\omega^{\mu\nu} J_{\mu\nu} + a^\mu E_\mu)\right) \exp\left(-\frac{i}{\hbar} t^\rho E_\rho\right) SO(1,3) = \cdots$$

$$\cdots = \exp\Bigg(-\frac{i}{\hbar}\Big(\underbrace{t^\rho E_\rho}_{\text{original } t^\mu \text{ part}} + \underbrace{(\omega^\mu{}_\rho t^\rho + a^\mu) E_\mu}_{\text{infinitesimal change}}\Big)\Bigg) SO(1,3).$$

$$\Longrightarrow \begin{pmatrix} dt^\mu \\ 0 \end{pmatrix} = \begin{pmatrix} \omega^\mu{}_\rho & a^\mu \\ 0 & 0 \end{pmatrix} \begin{pmatrix} t^\rho \\ 1 \end{pmatrix} = \begin{pmatrix} \omega^\mu{}_\rho t^\rho + a^\mu \\ 0 \end{pmatrix}.$$

# Another Look at our Alternative History

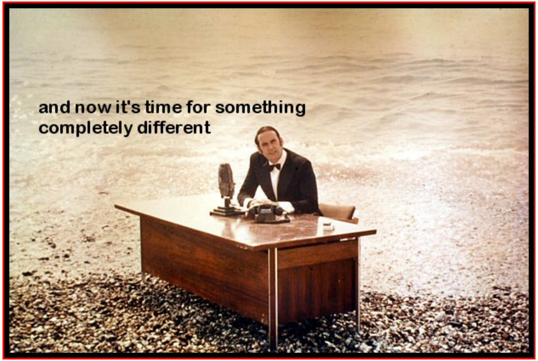


# On the Road from Kinematics to Dynamics

**Question:** We have an idea where things live. What else can we do?

That's a complicated question with many of the details yet to be determined... but here's a rough outline of what seems to have worked so far [2]:

- Given the coset space representation described above, one can define "Perelomov's generalized coherent states." These are the "most classical" states, in a certain sense), and allow one to reason more clearly about this model of physical space.
- Obtain a wavefunction description of this in terms of the "Heisenberg ring" of operators.
- Introduce the twisted convolution product; of these operators, take symplectic Fourier transform, define "Moyal bracket," etc. to get dynamics.


and now it's time for something
completely different

## Part II: The Necessity of Entanglement

**Rough Outline:**

A notion of *leaks* is introduced into arbitrary *process theories*. Upon applying this construction to the relevant process theory describing quantum theory, one finds that this is *leak free*. On the other hand, classical theory is *maximally leaking*.

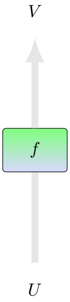
This can already be interpreted as giving an argument for the uniqueness of quantum theory in the vast landscape of available process theories related to classical theory. But even more can be said!

There is a construction, the *leak construction*, that introduces new leaks into a given theory. In the case of quantum theory, this construction is nothing more than *decoherence*. It turns out: any theory that decoheres to classical theory must contain entangled states. Entanglement is *necessary* for theories generalizing classical theory!

# Basics of Process Theories I

## The Building Blocks

**Physical Systems:** $U$ | **Physical Processes:** $f$

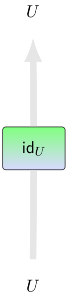




⋆ Here, we are thinking of "time" going up.

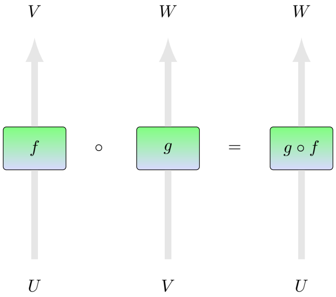
# Basics of Process Theories II

## Properties of Systems and Processes 1

$\forall$ systems $U$, *doing nothing* is a process



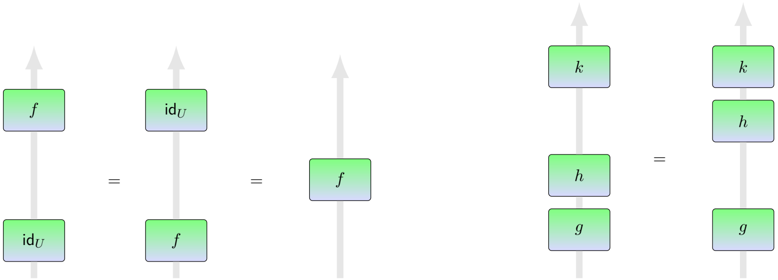

Processes happening in series is a thing




⋆ Typically, processes in series are written *vertically*.

# Basics of Process Theories II

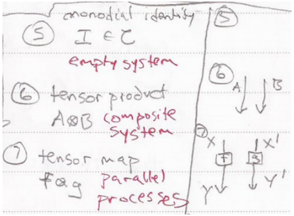
## Properties of Systems and Processes 2

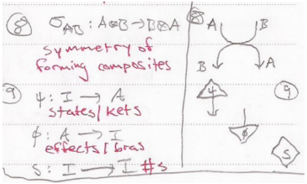


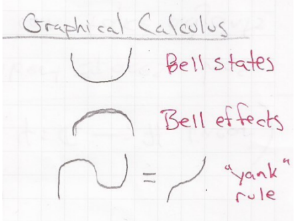

⋆ Together, these properties say that {Systems + Processes} forms a *category*.

# Basics of Process Theories III

## Additional Gadgets



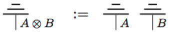




And more...

$\star$ This extra structure turns {Systems + Processes} into a *symmetric, monodial* $\dagger$*-category*.

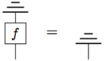
# Leaks in Process Theories [6]

We also make the natural assumption that discarding effects compose:




(1)

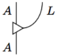
A process $f$ is causal if we have:




(2)

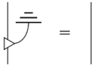
and a theory is causal if all of the processes of the theory are causal. Therefore, except for the fact that it composes, discarding is not subject to any defining constraints. In a sense, its behaviour is entirely implicit within its role within the defining equation of causality. In particular, by Equation (2) where $f$

**Definition 3.1.** *A leak is a process:*




(4)

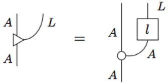
*which has discarding as a right counit, that is:*




(5)

# Example: Classical Leaks [6]

**Proposition 5.1.** *All classical leaks are of the form:*



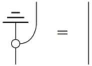

(12)

*where $l$ is any causal classical process.*

For classical probability theory, copying of support elements provides a leak:

 $: X \to X \times X :: x \mapsto (x, x)$

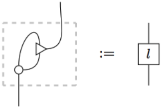
since if we discard a copy, we are back with what we started off with. In fact, strictly speaking, what we are dealing with here is not a copying operation since while it copies pure classical states, it does not do that for impure ones. What it is instead is broadcasting, that is besides Equation (5), discarding is also a left counit for the leaking process:



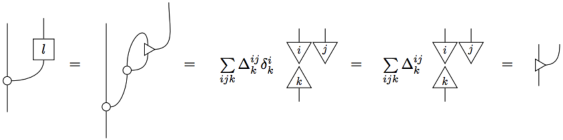

(6)

# Example: Classical Leaks [6]

*Proof.* First, let us define, using the non-causal "cap" of Example 2.5:




Then, we can check that Equation (12) is indeed satisfied:




□

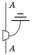
⋆ Similarly, one can show that all quantum leaks are *constant*.

# The Leak Construction [6]

**Theorem 6.1.** *Given any process theory and for each system a causal process:*

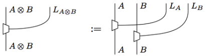

(14)

*which is such that the following process is idempotent:*




(15)

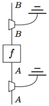
*and which are chosen coherently for composite systems:*




(16)

*we can construct a new process theory in which each process (14) is a leak for the system $A$. This construction goes as follows:*

- *systems stay the same;*
- *one restricts processes to those of the form:*




(17)

# Decoherence [6]

## The Leak Construction Applied to Quantum Theory

**Example 6.3** (Decoherence)**.** *The leak construction for the pre-leak:*

$$: \mathcal{B}(\mathcal{H}) \to \mathcal{B}(\mathcal{H} \otimes \mathcal{H}) :: |i\rangle\langle i| \mapsto |i\rangle\langle i| \otimes |i\rangle\langle i|$$

*applied to the process theory of quantum processes (i.e., Example 2.3), we obtain classical probability theory (i.e., Example 2.2).*

**Theorem 6.5.** *The leak construction applied to quantum processes (i.e., Example 2.3) gives all C*-algebras and C*-algebras only.*

Therefore, despite the weak structure of a leak, for the specific case of quantum theory, we obtain precisely the C*-algebras via the leak construction. This leads one to contemplate the view that the operational essence of (finite dimensional) C*-algebras is entirely captured by leaks and that the additional structure of C*-algebras is merely an artefact of the Hilbert space representation.

# An Application [7]

## Entanglement is Necessary

**RESULTS**

We are now in a position to prove our main result. If a theory can decohere to classical theory and does not have entanglement, then the original systems must be composites including a classical system, $\Omega_S = \Delta_N \otimes \Omega_f$, and the decoherence map simply discards any non-classical subsystems. More succinctly,

*Theories with non-trivial decoherence must have entangled states.*

謝謝！

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Part I

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Part II

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